

## Example Problem 2

### Aqueous flow to a well in a confined multi-layer system

**Abstract:** *This test case illustrates flow to a well in a confined multi-layer system, where two identical aquifers (upper and lower) are separated by an aquitard. The well produces only from the lower aquifer, where it is fully penetrating. This problem is known in the literature as the leaky aquifer problem. The user is introduced to a two-dimensional domain, a cylindrical coordinate system, and Neumann boundary conditions.*

#### 2.1 Problem Description

The input file for this problem is presented in section 2.3. Two identical aquifers, confined, horizontal, homogeneous, isotropic, 5 m thick, with hydraulic conductivity of  $2 \times 10^{-5}$  m/s and a storage coefficient of  $10^{-4}$ , are separated by an aquitard. The aquitard has a thickness of 10 m, hydraulic conductivity of  $10^{-8}$  m/s, and storage coefficient of  $8 \times 10^{-4}$ . Figure 2.1 shows a sketch of the conceptual model. A pumping well penetrates through to the lower aquifer. The well is only open to the full thickness of the lower aquifer, and it is sealed to the aquitard and upper aquifer. In addition, it is assumed the well is of small diameter, and there are no head losses due to well construction. The well is pumped at a constant rate  $Q$ , for  $t > 0$ . Uniform head prevails over the entire domain at  $t = 0$ , i.e. zero drawdown, and no drawdown is allowed at a radial distance of 10,000 m for  $t > 0$ .

The governing equation for transient flow in radial coordinates is

$$\frac{S_s}{K} \frac{\partial h}{\partial t} - \frac{\partial^2 h}{\partial r^2} - \frac{1}{r} \frac{\partial h}{\partial r} = 0 \quad 2.1$$

where  $S_s$ ,  $K$ ,  $h$ ,  $t$ , and  $r$ , are the specific storage coefficient, hydraulic conductivity, hydraulic head, time, and radial distance from the well, respectively.

A STOMP simulation with a duration of 100 years, an initial time-step of 36 s, and a time-step growth factor of 1.414 is shown in section 2.3. The simulation was started with uniform head initial conditions (hydrostatic pressure). Pumping at the well was implemented via a Neumann boundary condition, while a Dirichlet boundary condition was prescribed at  $r = 10,000$  m. The Boundary Conditions Card reads:

```
#-----
~Boundary Conditions Card
#-----
2,
west,neumann,
1,1,1,1,1,5,1,
0,d,-5.02929620e-05,m/s,
east,hydraulic gradient,
13,13,1,1,1,20,1,
0,d,684115,Pa,
```

The initial conditions of the problem were such that a uniform aqueous phase head of 60.0 m of water was imposed on the system. To obtain this initial distribution, the following Initial Conditions Card was imposed:

```
#-----
~Initial Conditions Card
#-----
Gas Pressure,Aqueous Pressure,
1,
Aqueous Pressure,684115,Pa,,,,,-9793.5,1/m,1,13,1,1,1,20,
```

Simulation of this problem requires a closely spaced grid near the well, in order to accurately represent the steep head gradient caused by pumping. Thus, a grid with progressively finer spacing close to the well is used in the radial direction. Uniform grid spacing is used to discretize the vertical direction. The resulting grid has 13 nodes in the radial direction and 20 nodes in the vertical direction. The radial symmetry of the problem allows the use of the cylindrical grid feature of STOMP. An  $18^\circ$  arc is used for the third dimension. Since the third dimension is not active, this angle is arbitrary. The Grid Card for this problem is:

```

#-----
~Grid Card
#-----
Cylindrical,
13,1,20,
1.0,m,1.195,m,2.365,m,3.955,m,7.285,m,12.715,m,22.885,m,40.315,m,72.085,m,127.915,m,
504.485,m,1495.515,m,4828.485,m,10000,m,
0.,deg,18.,deg,
0,m,20@1.0,m,

```

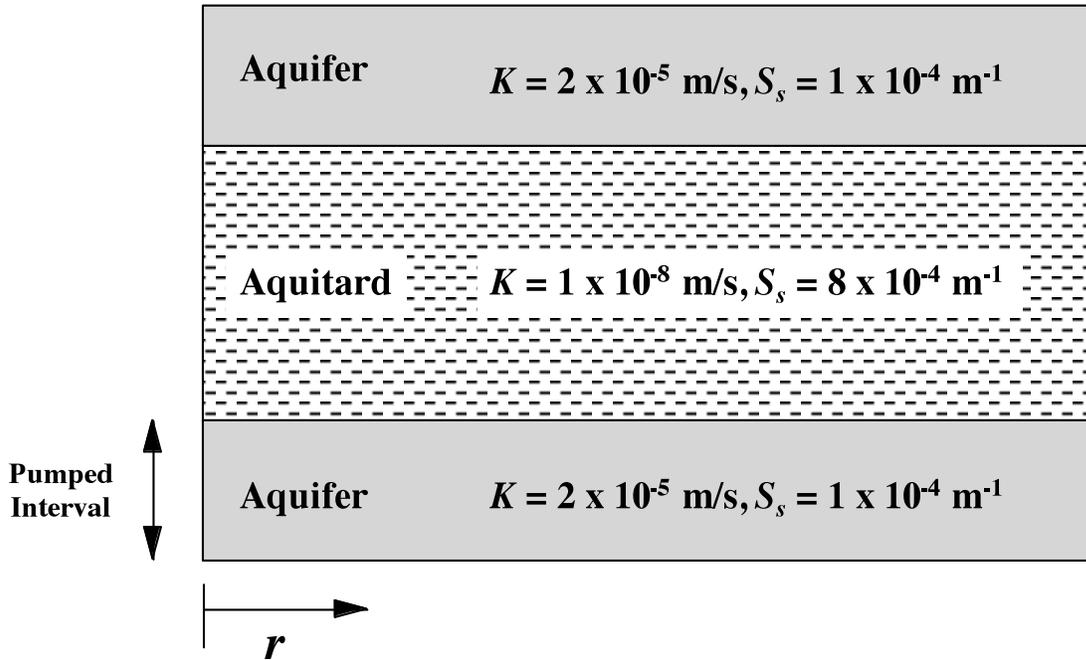
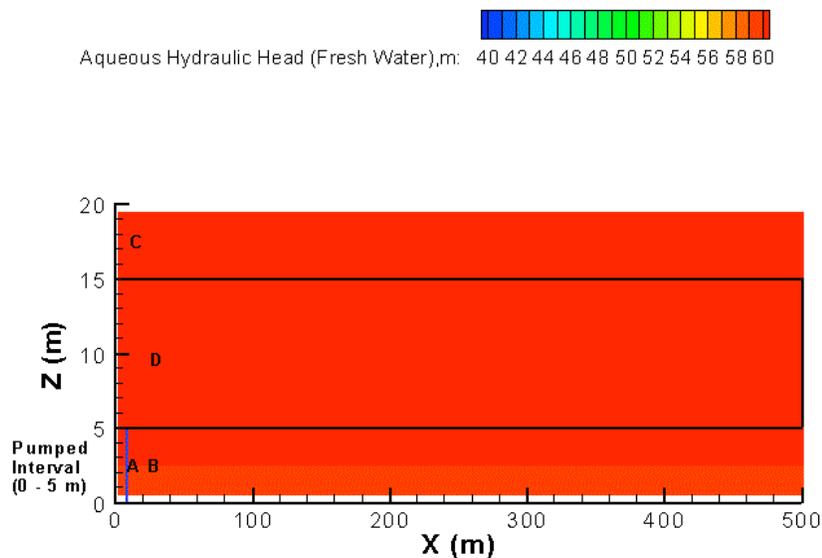


Figure 2.1 Schematic sketch of two-aquifer problem.

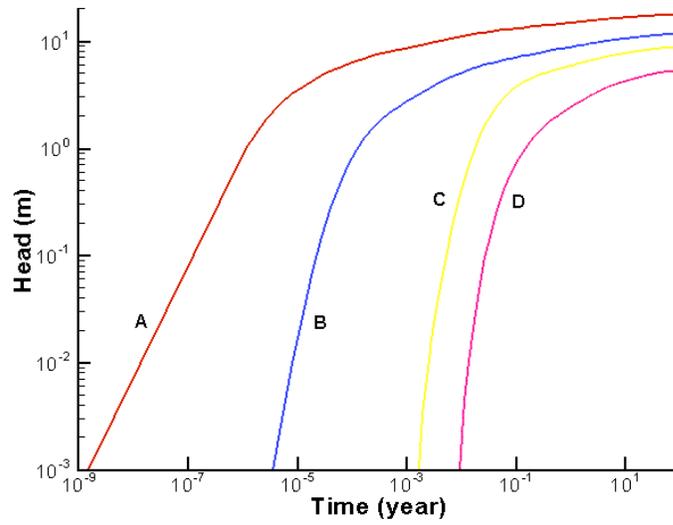
## 2.2 Exercises

1. Explain why the imposed initial conditions for the aqueous phase result in a pressure head of 59.5 m at the base nodes.
2. Compute the water production rate for the well in  $\text{m}^3/\text{day}$ .
3. Run the simulation using an appropriate executable and input file.
4. Compare the data in the *surface* file with the number computed in Exercise 2.

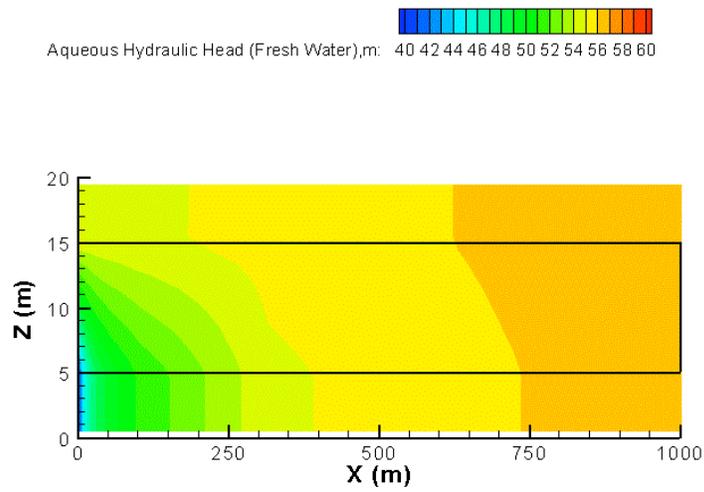
5. View the *output* file and plot the aqueous phase heads at the 4 reference nodes as a function of time. The location of the reference nodes are shown in Figure 2.2. Compare the produced plot with the results shown in Figure 2.3 and explain the differences.
6. Manipulate the final *plot* file to produce a plot that shows the head distribution for the whole or part of the domain. The plot has to look similar to Figure 2.4.
7. Increase the hydraulic conductivity of the aquitard to equal that of the two aquifers. Run the simulation and make a plot of the final aqueous phase head distribution. Compare the results with the plot made in Exercise 6. Reset the hydraulic conductivity value in the in the *input* file.
8. Impose an anisotropy ratio of 10:1 in the aquitard only. Run the simulation and make a plot of the final aqueous phase head distribution. Compare the results with the plots made in Exercises. 6 and 7. Reset the hydraulic conductivity value in the in the *input* file.
9. Instead of using the Neumann boundary condition, create a Source Card to obtain similar results. Use the User's Guide and the first example on Page B. 185 for guidance. Run the simulation and compare the results.



**Figure 2.2** Location of specified reference nodes and initial aqueous head distribution.



**Figure. 2.3.** Drawdown at location A, B, C, D.



**Figure 2.4.** Final aqueous head distribution.

## 2.3 Input File

```
#-----  
~Simulation Title Card  
#-----  
1,  
STOMP Tutorial Problem 2,  
Mart Ostrom/ Mark White,  
PNNL,  
June 03,  
15:00,  
4,  
Two aquifer problem (Segol 1994, p. 423-432),  
Two aquifers (5 m thick) separated by an aquitard (10 m thick),  
Pumping well is screened in the lower aquifer only,  
No drawdown allowed on other end,  
  
#-----  
~Solution Control Card  
#-----  
Normal,  
Water,  
1,  
0,s,100,yr,36,s,50,d,1.414,24,1.e-6,  
10000,  
,  
  
#-----  
~Grid Card  
#-----  
Cylindrical,  
13,1,20,  
1.0,m,1.195,m,2.365,m,3.955,m,7.285,m,12.715,m,22.885,m,40.315,m,72.085,m,127.915,m,  
504.485,m,1495.515,m,4828.485,m,10000,m,  
0.,deg,18.,deg,  
0,m,20@1.0,m,  
  
#-----  
~Rock/Soil Zonation Card  
#-----  
3,  
Aquifer1,1,13,1,1,16,20,  
Aquitard,1,13,1,1,6,15,  
Aquifer2,1,13,1,1,1,5,  
  
#-----  
~Mechanical Properties Card  
#-----  
Aquifer1,2650,kg/m^3,0.30,0.30,1.e-04,1/m,  
Aquitard,2650,kg/m^3,0.43,0.43,8.e-04,1/m,  
Aquifer2,2650,kg/m^3,0.30,0.30,1.e-04,1/m,  
  
#-----  
~Hydraulic Properties Card  
#-----  
Aquifer1,2.e-05,hc m/s,,,2.e-05,hc m/s,  
Aquitard,1.e-08,hc m/s,,,1.e-08,hc m/s,  
Aquifer2,2.e-05,hc m/s,,,2.e-05,hc m/s,
```

```

#-----
~Saturation Function Card
#-----
Aquifer1,van Genuchten,0.133,1 / cm,1.88,0.268,,
Aquitard,van Genuchten,0.133,1 / cm,1.88,0.268,,
Aquifer2,van Genuchten,0.133,1 / cm,1.88,0.268,,

#-----
~Aqueous Relative Permeability Card
#-----
Aquifer1,Mualem,,
Aquitard,Mualem,,
Aquifer2,Mualem,,

#-----
~Initial Conditions Card
#-----
Gas Pressure,Aqueous Pressure,
1,
Aqueous Pressure,684115,Pa,,,,,-9793.5,1 / m,1,13,1,1,1,20,

#-----
~Boundary Conditions Card
#-----
2,
west,neumann,
1,1,1,1,5,1,
0,d,-5.02929620e-05,m / s,
east,hydraulic gradient,
13,13,1,1,1,20,1,
0,d,684115,Pa,

#-----
~Output Options Card
#-----
4,
3,1,3,
7,1,3,
3,1,18,
7,1,10,
1,1,yr,m,deg,6,6,6,
3,
aqueous saturation,,
aqueous pressure,Pa,
aqueous hydraulic head,m,
5,
0.5,d,
1,d,
1,yr,
10,yr,
50,yr,
8,
no restart,,
aqueous hydraulic head,m,
aqueous pressure,Pa,
aqueous saturation,,
XNC aqueous volumetric flux,m / hr,
ZNC aqueous volumetric flux,m / hr,

```

X aqueous volumetric flux,m/hr,  
Z aqueous volumetric flux,m/hr,

#-----  
~Surface Flux Card  
#-----  
2,  
aqueous volumetric flux,m<sup>3</sup>/day,m<sup>3</sup>,west,1,1,1,1,1,5,  
aqueous volumetric flux,m<sup>3</sup>/day,m<sup>3</sup>,east,13,13,1,1,1,20,

## 2.4 Solutions to Selected Exercises

### Exercise 1

Initial conditions for aqueous pressure for this problem are specified by specifying a base pressure and pressure gradient on the *Initial Conditions Card*. The base pressure is given as 684115 Pa. This pressure is  $684,115 - 101,325 = 582,780$  Pa larger than atmospheric pressure. The vertical pressure gradient of  $-9793.52$  is equivalent to uniform total head conditions (i.e., hydrostatic conditions) for an aqueous density of  $998.32 \text{ kg/m}^3$  (i.e., pure water at 20 C). Dividing  $582,780$  Pa by the  $9793.52 \text{ Pa/m}$  gradient yields a pressure head of  $59.5$  m.

### Exercise 2

The water production rate is  $136.5 \text{ m}^3/\text{day}$  ( $1502.6 \text{ gal/hr}$ ), calculated using the specified boundary flux of  $5.0292962\text{e-}05 \text{ m/s}$  and the well geometry, as

$$5.0292962 \times 10^{-5} \text{ m/s} * 2\pi * 1 \text{ m} * 5 \text{ m} = 1.58 \times 10^{-3} \text{ m}^3/\text{s} = 136.5 \text{ m}^3/\text{day}$$

where, the screened interval area equals  $31.416 \text{ m}^2$ .

### Exercises 3 and 4

The surface flux file reports a steady-flow flow rate into the well of  $6.8256 \text{ m}^3/\text{day}$ , which can be converted to a well production rate, as

$$6.8256 \text{ m}^3/\text{day} * 18.0^\circ/360.0^\circ = 136.5 \text{ m}^3/\text{day}$$

which agrees with the prescribed water production rate for the well.

### Exercise 5

The plot of hydraulic head at each location as a function of time, as shown in Figure 2.4, shows decreasing head with time. This plot differs from the results shown in Figure 2.3, as it shows hydraulic head drawdown, calculated as the initial hydraulic head (i.e., 60 m) less hydraulic head.

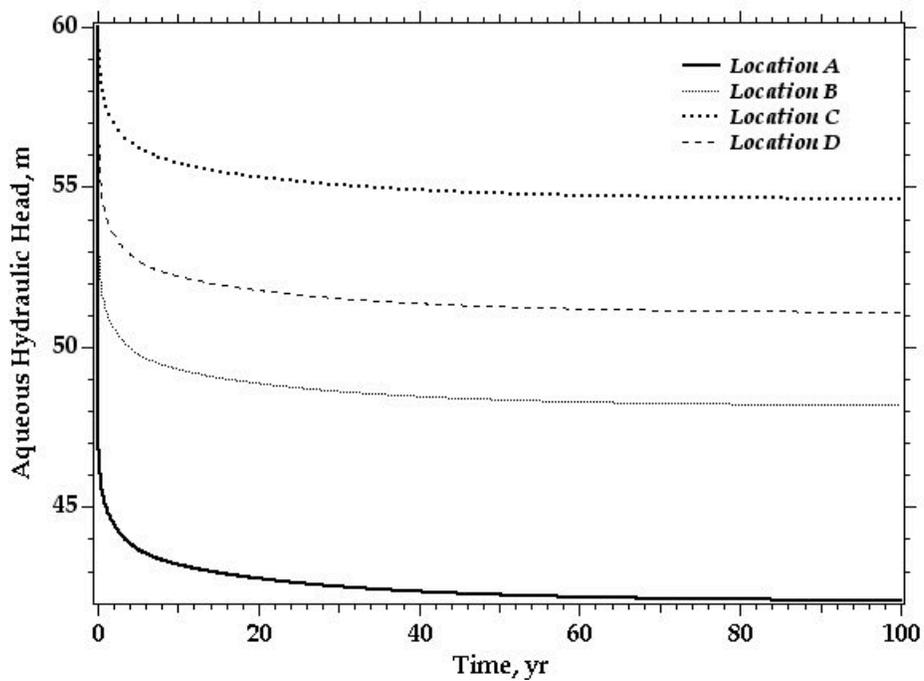
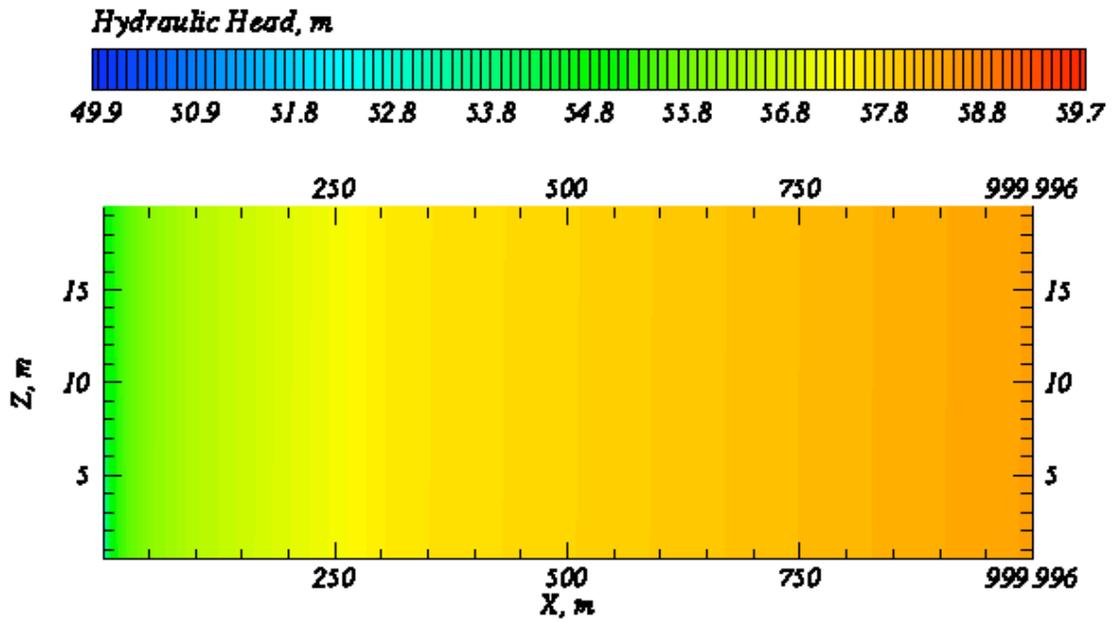


Figure 2.4. Hydraulic head in response to pumping

### Exercise 7

Increasing the hydraulic conductivity of the aquitard to the value of the aquifers reduces the overall flow resistance and homogenizes the aquifer. The lower resistance reduces the gradient in hydraulic head and the homogenization flattens the head contours, as shown in Figure 2.5.



**Figure 2.5.** Contours of hydraulic head in response to pumping (homogeneous aquifer).

### Exercise 8

Increasing the horizontal hydraulic conductivity of the aquitard by a factor of 10, reduces the horizontal flow resistance of the aquitard, but not enough to significantly change the response in hydraulic head, as shown in Figure 2.6.

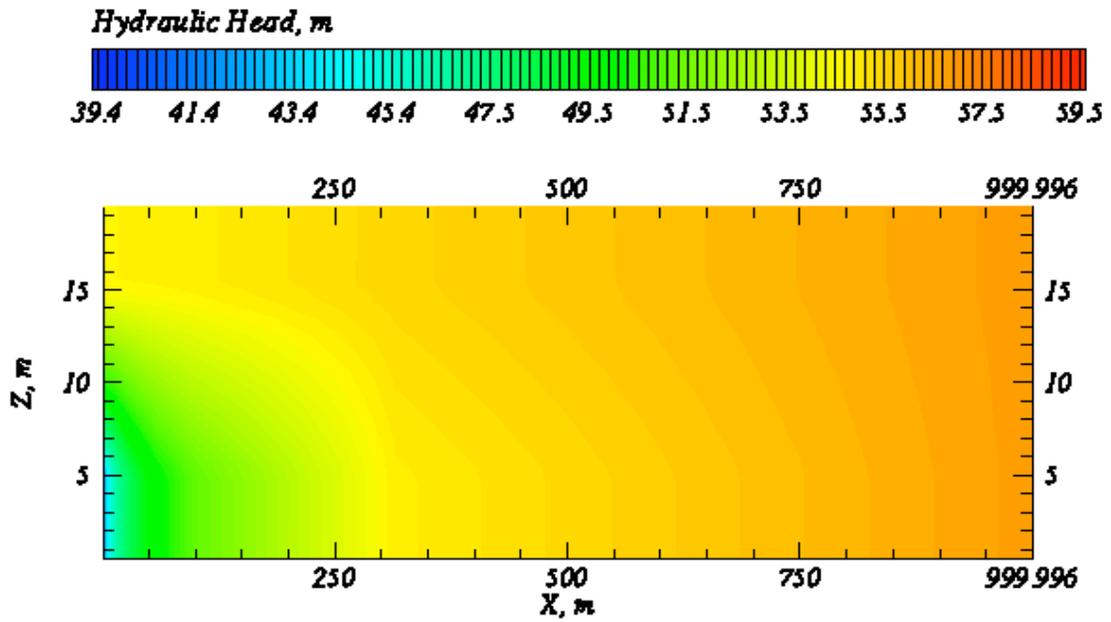


Figure 2.6. Contours of hydraulic head in response to pumping (10:1 anisotropy ratio).

### Exercise 9

~Source Card  
 1,  
 Aqueous Volumetric,1,1,1,1,1,5,1,  
 0,d,-1.58e-05,m<sup>3</sup>/s,