Example Problem 4
Salt-water intrusion and density-driven flow:
Henry's Problem

Abstract: *Henry's problem addresses the steady-state solution of a diffused salt-water wedge within a confined aquifer balanced against a flowing fresh-water field. Fresh water enters the confined aquifer at a constant rate from a hypothetical inland boundary and discharges into a hypothetical coastal boundary. Salt water from the coastal boundary advances and mixes with the discharging fresh water. The user is introduced to modifications to the input file that allow appropriate transport of density and viscosity altering dissolved components. The Water-Salt mode (STOMP-WS) is used for this application.*

4.1 Problem Description

This application was chosen to demonstrate the coupled flow and salt transport capabilities of the STOMP simulator. Although these capabilities have been specifically written for salt-water brines, other solutes could be considered by changing the algorithms for computing the brine properties (e.g., density and viscosity).

Henry's problem addresses the steady-state solution of a diffused salt-water wedge within a confined aquifer balanced against a flowing fresh-water field. Fresh water enters the confined aquifer at a constant rate from a hypothetical inland boundary and discharges into a hypothetical coastal boundary. Salt water from the costal boundary advances and mixes with the discharging fresh water. Because both the inland and costal boundary conditions are invariant a steady-state condition is reached, which balances the intruding sea-water wedge against the fresh-water flow field. Henry (1964a; 1964b) published an analytical solution to this problem in a U.S. Geological Survey publication, and the problem has henceforth become a classical test for numerical simulators with solute dependent density capabilities. Unfortunately, no other numerical method has been able to successful in duplicating Henry's solution, which accordingly resulted in some doubt about its validity. Ségol (1994) revisited Henry's solution and noted several discrepancies in the published solution. Ségol's revisited solution to this classical problem shows close agreement with the numerical solution of Voss and Souza (1987).

Henry's problem involves a two-dimensional rectangular domain with no flow conditions along the top and bottom boundaries to simulate a confined aquifer of infinitesimal width, as shown in Figure 4.1. This problem description follows that developed by Voss and Souza (1987) from Henry's original formulation. The rectangular domain has dimensions of 2 m in the horizontal direction and 1 m in the vertical direction, which is aligned with the gravitational vector. The computational grid comprises 200 square nodes of uniform size. A constant fresh-water flux (Neumann condition) is imposed on the inland (west) boundary; whereas, a hydrostatic pressure boundary (hydraulic gradient condition) of salt water is imposed on the costal (east) boundary. Parameters used in this simulation are consistent with the non-dimensional parameters chosen by Henry (1964a; 1964b). Initially the aquifer was filled with freshwater under hydrostatic conditions. The pressure boundary conditions on the costal boundary were hydrostatic conditions for sea water. Henry's problem was solved with the STOMP simulator by executing from fresh-water hydrostatic conditions in the aquifer until steady state conditions were reached. The time step acceleration factor of 1.25 allows the user to over specify the time required to reach steady- state conditions without excessive execution time costs. As the simulation approaches steady-state conditions the number of Newton-Raphson iterations will diminish to one, and all of the output variables will become invariant with time. Steady-state conditions for this problem were achieved roughly after 1 day of simulation time. The relatively small initial time step of 1.0 s was chosen to prevent convergence failures during the first time step.

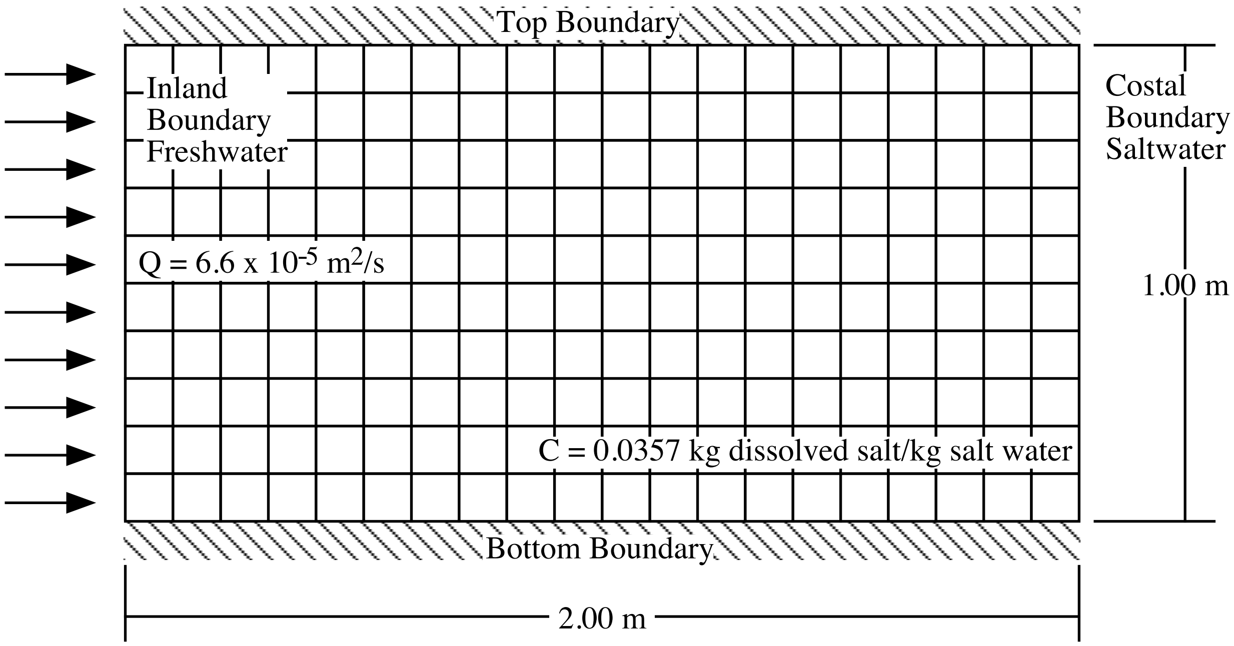

Figure 4.1 Henry's problem and computational grid

The salt interactions with the aqueous phase and the porous medium are specified in the Salt Transport Card.

```
#-------------------------------------------------------------------
~Salt Transport Card
#-------------------------------------------------------------------
Henry Sodium Chloride,
Constant,18.86e-6,m^2/s,
Porous Medium,0.0,m,0.0,m,
```

Two modifications, involving the computation of salt-water properties, were made to the source coding of the STOMP simulator to execute this application. Salt-water density in the STOMP simulator is normally computed using the function of Leijnse (1992), according to Eq. 4.1. For our application this function was replaced with the one specified by Henry, as shown in Eq. 4.2. The variation in salt-water viscosity was ignored in Henry's problem; therefore, the expression for salt-water viscosity was modified from the function of Leijnse (1992), according to Eq. 4.3 to that shown in Eq. 4.4. The simulator is prompted to use the special density and viscosity equations for this problem by including the word 'Henry' in the first line of the Salt Transport Card. The second line shows the salt diffusion coefficient in the aqueous phase, and the third line the dispersivity values. The boundary conditions for the salt equation are given in the Boundary Conditions Card. In this case, constant aqueous salt concentrations are prescribed at both the west and east boundaries.

$$\rho_\ell^s = \rho_\ell^w \exp\left(0.7\omega_\ell^s\right) \qquad 4.1$$

$$\rho_\ell^s = \rho_\ell^w + 0.6829 C_\ell^s \qquad 4.2$$

$$\mu_\ell^s = \mu_\ell^w \left[1.0 + 1.85\omega_\ell^s + 4.1\left(\omega_\ell^s\right)^2 + 44.5\left(\omega_\ell^s\right)^3\right] \qquad 4.3$$

$$\mu_\ell^s = \mu_\ell^w \qquad 4.4$$

References

Henry H.R. 1964a. Effects of Dispersion on Salt Encroachment in Coastal Aquifers. Water-Supply Paper 1613-C, Sea Water in Coastal Aquifers: C71-84, U.S. Geological Survey.

Henry H.R. 1964b. Interfaces between salt water and fresh water in coastal aquifers. Water-Supply Paper 1613-C, Sea Water in Coastal Aquifers: C35-70, U.S. Geological Survey.

Leijnse A. 1992. *Three-Dimensional Modeling of Coupled Flow and Transport in Porous Media*, Ph.D., University of Notre Dame, Notre Dame, Indiana.

Ségol G. 1994. *Classic Groundwater Simulations: Proving and Improving Numerical Models*. Prentice-Hall, Englewood Cliffs, New Jersey.

Voss C.I. and W.R. Souza. 1987. "Variable density flow and solute transport simulation of regional aquifers containing a narrow freshwater-saltwater transition zone." *Water Resources Research*, 23:1851-1866.

4.2 Exercises

1. Run the simulation and produce a plot of the salt distributions after 10 days.
2. Reduce the west Neumann aqueous phase inflow rate by a factor 10. Run the simulation and make a plot of the salt distribution at t = 10 days. Compare graph with the results obtained in Exercise 1. Restore input file.
3. Reduce the salt diffusion coefficient by a factor 10. Run the simulation and make a plot of the salt distribution at t = 10 days and compare graph with the results obtained in Exercises 1 - 2 Restore input file.
4. Add an aqueous volumetric source at node 15,1,1. Allow the source to extract water with a rate of 10 l/min. Run the simulation and make a plot of the salt distribution at t = 10 days and compare the graph with the results obtained in Exercises 1 - 4. If the differences are small compared to the base case, increase the pumping rate until a change is observed.

4.3 Input File

```
#-------------------------------------------------------------------
~Simulation Title Card
#-------------------------------------------------------------------
1,
STOMP Tutorial Problem 4,
Mart Oostrom/Mark White,
PNNL,
June 03,
15:00,
2,
Henry's Problem for Salt Water Intrusion,
Classic test problem for simulators with water and salt equations,

#-------------------------------------------------------------------
~Solution Control Card
#-------------------------------------------------------------------
Normal,
Water-Salt,
1,
0,yr,10,d,1,s,1,d,1.25,8,1.e-6,
1000,
,

#-------------------------------------------------------------------
~Grid Card
```

```
#-------------------------------------------------------------
Uniform Cartesian,
20,1,10,
10,cm,
10,cm,
10,cm,

#-------------------------------------------------------------
~Rock/Soil Zonation Card
#-------------------------------------------------------------
1,
Porous Medium,1,20,1,1,1,10,

#-------------------------------------------------------------
~Mechanical Properties Card
#-------------------------------------------------------------
Porous Medium,,,0.35,0.35,,,Constant Diffusion,1.0,

#-------------------------------------------------------------
~Hydraulic Properties Card
#-------------------------------------------------------------
Porous Medium,1.020408e-9,m^2,,,1.020408e-9,m^2,

#-------------------------------------------------------------
~Saturation Function Card
#-------------------------------------------------------------
Porous Medium,van Genuchten,0.2,1/cm,1.8,0.0,,

#-------------------------------------------------------------
~Aqueous Relative Permeability Card
#-------------------------------------------------------------
Porous Medium,Mualem,,

#-------------------------------------------------------------
~Salt Transport Card
#-------------------------------------------------------------
Henry Sodium Chloride,
Constant,18.86e-6,m^2/s,
Porous Medium,0.0,m,0.0,m,

#-------------------------------------------------------------
~Initial Conditions Card
#-------------------------------------------------------------
Gas Pressure,Aqueous Pressure,
1,
Aqueous Pressure,121325,Pa,,,,,-9793.5331,1/m,1,20,1,1,1,10,

#-------------------------------------------------------------
~Boundary Conditions Card
#-------------------------------------------------------------
2,
west,neumann,aqueous conc,
1,1,1,1,1,10,1,
0,s,6.6e-05,m/s,0.0,kg/m^3,
east,hydraulic gradient,aqueous conc,
20,20,1,1,1,10,1,
0,s,121557.98,Pa,36.5921,kg/m^3,
```

```
#-------------------------------------------------------------
~Output Options Card
#-------------------------------------------------------------
10,
20,1,1,
20,1,3,
20,1,5,
20,1,7,
20,1,10,
15,1,1,
15,1,3,
15,1,5,
15,1,7,
15,1,10,
1,1,d,m,6,6,6,
5,
salt aqueous concentration,kg/m^3,
aqueous viscosity,cp,
aqueous density,kg/m^3,
xnc aqueous volumetric flux,m/s,
znc aqueous volumetric flux,m/s,
2,
0.5,d,
1,d,
6,
no restart,,
salt aqueous concentration,kg/m^3,
aqueous viscosity,cp,
aqueous density,kg/m^3,
xnc aqueous volumetric flux,m/s,
znc aqueous volumetric flux,m/s,

#-------------------------------------------------------------
~Surface Flux Card
#-------------------------------------------------------------
4,
aqueous volumetric flux,m^3/day,m^3,east,20,20,1,1,1,5,
aqueous volumetric flux,m^3/day,m^3,east,20,20,1,1,6,10,
salt mass flux,kg/day,kg,east,20,20,1,1,1,5,
salt mass flux,kg/day,kg,east,20,20,1,1,6,10,
```

4.4 Solutions to Selected Exercises

Exercise 1

The steady-state solution to Henry's problem is shown in Figure 4.2. The salt water is moving in through diffusion from the east boundary. The density effects are clearly visible as the salt water tends to sink in the domain. The fresh water, moving from west to east, is diverted to flow over the salt water.

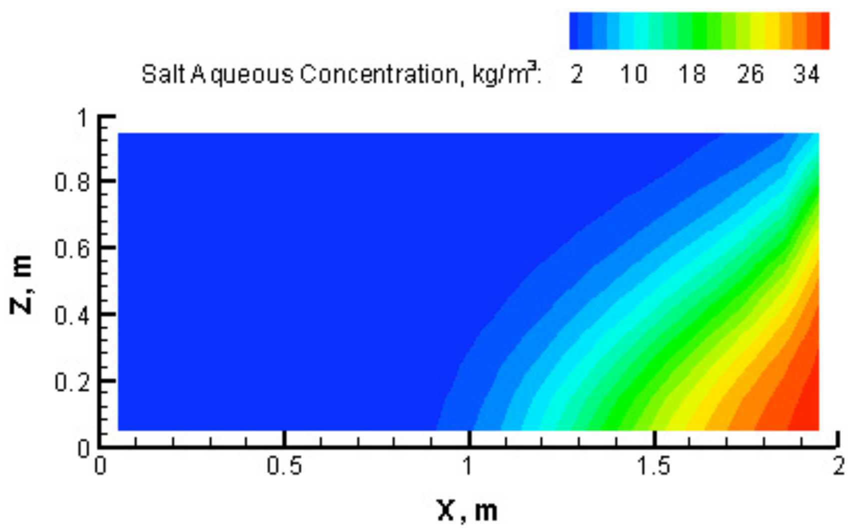

Figure 4.2 Salt concentrations for the standard Henry simulation at t = 10 days.

Exercise 2

Exercise 3

A reduction of the aqueous flux at the west boundary results in an increased diffusion of salt into the domain from the east boundary (Figure 4.3). The amount of salt that is transported into the domain is considerably more than the amount for the standard problem (Figure 4.2).

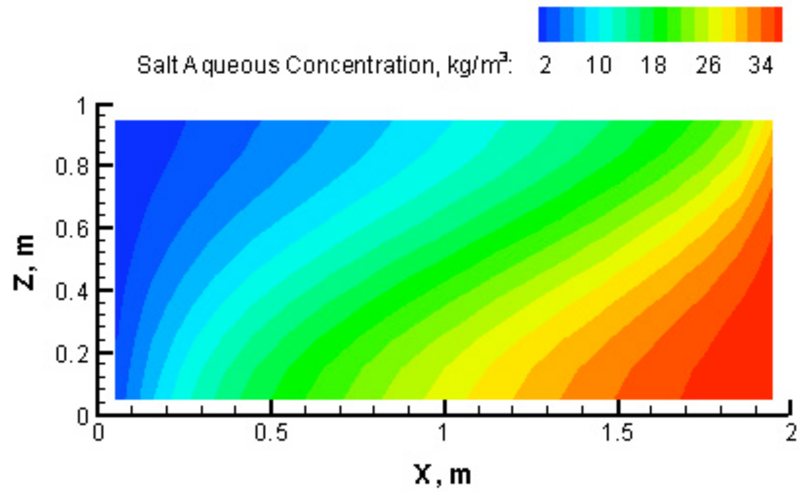

Figure 4.3 Salt concentrations following the reduced fresh-water inflow rate.

Exercise 4

A reduction in the diffusion coefficient causes decreased mixing of the salt with the fresh water (Figure 4.4. The sinking of the wedge is more pronounced as greater density gradients occur in the domain.

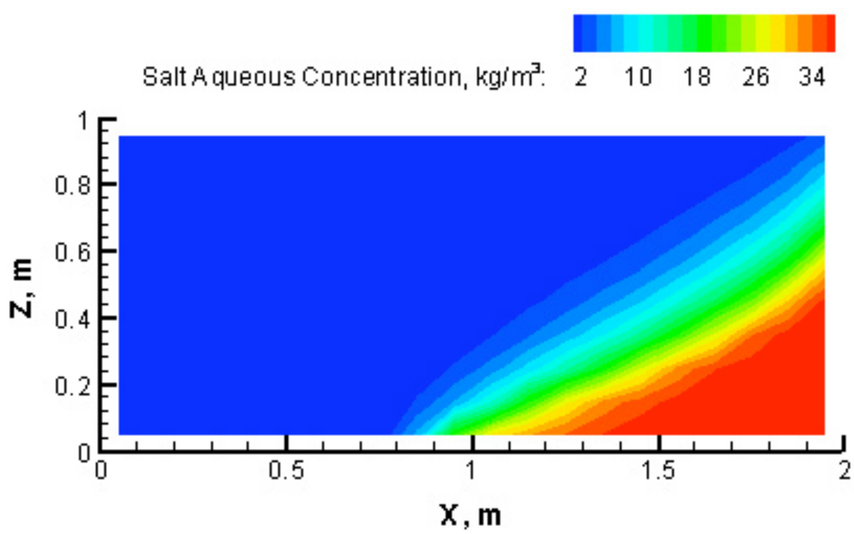

Figure 4.4 Salt concentrations as a result of a reduced salt diffusion coefficient.

Exercise 5

As a result of pumping, the salt wedge is reduced compared to the base case. The steady-state situation, depicted in Figure 4.6, occurs after less than one hour.

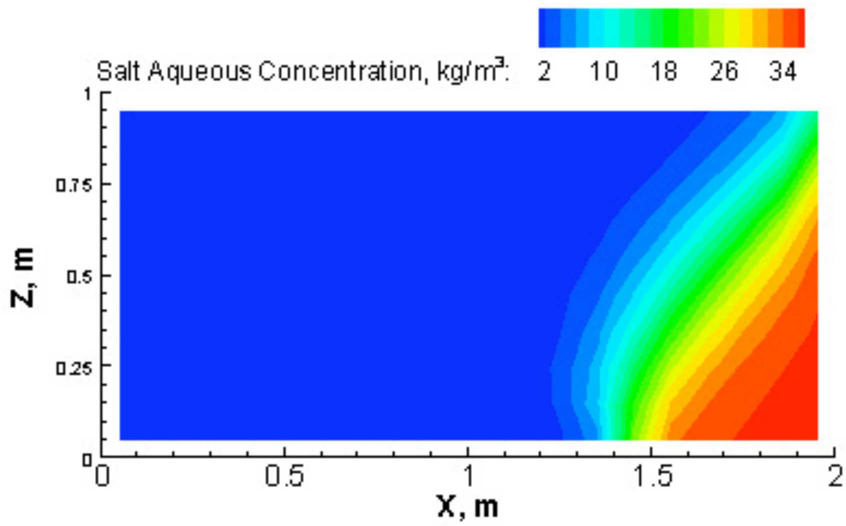

Figure 4.5 Salt concentrations resulting from well pumping at node 15, 1, 1, extracting 10 L/hour. The steady state situation is reached in about one hour.